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The method of the 'carry over' of integral transforms in non-linear mass and heat transfer problems

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Abstract—A simple approximate method for solving non-linear problems of non-stationary mass and heat transfer is suggested. The method guarantees accurate fulfillment of initial and boundary conditions, often leads to a correct asymptotic at large times and gives an accurate result for linear problems. Non-stationary heat transfer between a wall and a quiescent medium with the thermal conductivity coefficient being arbitrarily dependent on temperature is investigated. Non-stationary problems of mass transfer with volumetric and surface reactions of any order are investigated.

1. DESCRIPTION OF THE METHOD

THE MOST common way of solving linear problems is to use different integral transforms of the unknown function (Laplace–Carson, Mellin and Bessel transforms and others [1]) which can tentatively be written in the form

$$u = \mathbf{L} * c \tag{1}$$

where c is the unknown function (inverse image), $\mathbf{L}$ some integral operator, and u the image.

In a number of cases transform (1) can also be successfully used for an approximate analysis of non-linear problems by 'carrying over' the transform under the function sign according to the rule

$$\mathbf{L} * f(c) \simeq f(\mathbf{L} * c) = f(u) \tag{2}$$

where $f = f(c)$ is some non-linear function of the argument c. The validity region of the approximate operation (2) should be established separately in each specific case.

Further, the discussion will be limited to the study of the Laplace–Carson transform which is defined as

$$u = \mathbf{L} * c \equiv p \int_0^\infty e^{-p\tau}\, c \,d\tau \tag{3}$$

where p is the complex parameter.

Non-linear parabolic equations of the following form will be considered:

$$\frac{\partial c}{\partial \tau} = \Phi\left(x, c, \frac{\partial c}{\partial x}, \frac{\partial^2 c}{\partial x^2}\right) \tag{4}$$

with the initial and boundary conditions

$$\tau = 0, \quad c = \phi(x) \tag{5}$$

$$x = x_1,\ c = \psi_1(\tau); \quad x = x_2,\ c = \psi_2(\tau) \tag{6}$$

where Φ, ϕ, ψ_1, ψ_2 are arbitrary sufficiently smooth functions.

Applying the Laplace–Carson transform (3) to problem (4)–(6) and 'carrying' it under the sign of the function Φ according to rule (2) gives the approximate ordinary differential equation and boundary conditions for u

$$p[u - \phi(x)] = \Phi\left(x, u, \frac{du}{dx}, \frac{d^2u}{dx^2}\right) \tag{7}$$

$$x = x_1,\ u = \alpha_1(p); \quad x = x_2,\ u = \alpha_2(p) \tag{8}$$

where $\alpha_i(p) = \mathbf{L} * \psi_i$; $i = 1, 2$.

Having constructed the solution of problem (7), (8) and then applying the Laplace–Carson inverse transform to u [1], one can find function c.

The proposed approximate method leads to an exact solution for any linear problem. It always guarantees accurate fulfilment of initial and boundary conditions (5) and (6). Moreover, if the solution of problem (4)–(6) is stabilized at large times, i.e. there exists the solution of the stationary problem

$$\Phi\left(x, c, \frac{dc}{dx}, \frac{d^2c}{dx^2}\right) = 0 \tag{9}$$

$$x = x_1,\ c = A_1; \quad x = x_2,\ c = A_2$$

$$\left(A_i = \lim_{\tau\to\infty} \psi_i(\tau), \quad \text{where } i = 1, 2\right) \tag{10}$$

then the approach described gives a correct asymptotic result for $\tau \to \infty$.

The application of the 'carry-over' method of integral transforms will now be illustrated on specific examples which are of interest for the theory of mass and heat transfer. As usual, attention will be mostly

NOMENCLATURE

a	length scale (radius of a droplet, bubble)
C	concentration
C_s	concentration on the surface of a wall, droplet, bubble
C_0	concentration of the initial time instant
c	dimensionless concentration (determined in different ways)
D	coefficient of diffusion, may be dependent on concentration, $D(C)$
$\bar{D}(c)$	$D(C)/D(C_s)$
F_s	surface reaction rate, $F_s(C)$, for the nth-order reaction, K_sC^n
F_v	volumetric reaction rate, $F_v(C)$, for the nth-order reaction, K_vC^n
$f_s(c)$	dimensionless rate of surface reaction, $aF_s(C)/DC_0$, for the nth-order reaction, $f_s = k_s(1-c)^n$
$f_v(c)$	dimensionless rate of volumetric reaction, $a^2F_v(C)/DC_s$, for the nth-order reaction, $f_v = k_vc^n$
j	dimensionless diffusion flow on a wall, $-(\partial c/\partial x)_{x=0}$
K_s	constant of surface reaction rate
K_v	constant of volumetric reaction rate
k_s	dimensionless constant of the nth-order surface reaction rate, $aK_sC_0^{n-1}/D$
k_v	dimensionless constant of the nth-order volumetric reaction rate, $a^2K_vC_s^{n-1}/D$
n	order of reaction
Pe	Peclet number, $aU/D(C_s)$
Sh	Sherwood number, equation (46)
t	time
U	characteristic velocity of liquid
U_∞	translational flow velocity far from a droplet
u	concentration image, equation (3)
$\mathbf{v}$	dimensionless velocity of liquid
X	distance to a wall
x	X/a.

Greek symbols

β	ratio of droplet to surrounding medium dynamic viscosities, $\beta = 0$ corresponds to a gas bubble
τ	dimensionless time, tD/a^2 or $tD(C_s)/a^2$.

paid to the derivation of formulae for diffusion (heat) flows.

2. MASS TRANSFER IN THE CASE OF ARBITRARY DEPENDENCE OR THE DIFFUSION COEFFICIENT ON CONCENTRATION

Consider a non-stationary problem of mass transfer between a wall and a quiescent medium with the coefficient of diffusion D being arbitrarily dependent on the concentration C.

The concentration distribution is described by the non-linear equation

$$\frac{\partial c}{\partial \tau} = \frac{\partial}{\partial x}\bar{D}(c)\frac{\partial c}{\partial x} \tag{11}$$

under the following initial and boundary conditions:

$$\tau = 0,\ c = 0; \quad x = 0,\ c = 1; \quad x \to \infty,\ c \to 0. \tag{12}$$

Here, the dimensionless variables are expressed in terms of dimensional ones with the aid of the following relations:

$$c = \frac{C_0 - C}{C_0 - C_s}, \quad x = \frac{X}{a}, \quad \tau = \frac{tD(C_s)}{a^2}, \quad \bar{D}(c) = \frac{D(C)}{D(C_s)}$$

where C_0 is the concentration at the initial time instant, C_s the concentration on the wall, X the distance to the wall, t the time, and a the quantity which has the dimensions of length. The function $\bar{D}$ is introduced in such a way that the condition $\bar{D}(1) = 1$ is valid.

An analogous formulation is employed for the problem on heat transfer between a wall and a quiescent medium with the thermal conductivity coefficient being arbitrarily dependent on temperature.

Applying the Laplace–Carson transform to equation (11), initial and boundary conditions (12), carrying out the transform under the sign of the function $\bar{D}$, similarly to equation (7), one obtains

$$pu = \frac{d}{dx}\bar{D}(u)\frac{du}{dx} \tag{13}$$

$$x = 0,\ u = 1; \quad x \to \infty,\ u \to 0. \tag{14}$$

Taking into account the fact that the ordinary differential equation (13) is not explicitly dependent on x, its order can be reduced by the standard substitution

$$w = \frac{du}{dx}\left(\frac{d}{dx} = w\frac{d}{du}\right). \tag{15}$$

This yields

$$pu = w\frac{d}{du}[\bar{D}(u)w]. \tag{16}$$

It can be easily checked that the general solution of this equation has the form

$$2p\int_0^u u\bar{D}(u)\,du = [\bar{D}(u)w]^2 + B \tag{17}$$

where B is an arbitrary constant.

It follows from the boundary condition at the infinity (14) that $w = du/dx \to 0$ when $x \to \infty$. Therefore, changing over to the limit $x \to \infty$ in expression (17) (the fact corresponds to $u = w = 0$), it can be found that the constant $B = 0$.

With the above taken into account, equation (17) will be rewritten as

$$\bar{D}(u)\frac{du}{dx} = -\left[2p\int_0^u u\bar{D}(u)\,du\right]^{1/2}. \tag{18}$$

The formula will now be introduced to calculate the dimensionless diffusion flow onto the wall $j = -\bar{D}(1)(\partial c/\partial x)_{x=0}$. For this, $x = 0$ will be substituted into both parts of equality (18); according to the first boundary condition (14) this corresponds to the value $u = 1$. Using the inverse Laplace–Carson transform and taking into account the property $\bar{D}(1) = 1$, the approximate expression sought for the flow can be obtained

$$j = \left[\frac{2}{\pi\tau}\int_0^1 c\bar{D}(c)\,dc\right]^{1/2}. \tag{19}$$

This formula gives an exact result for the constant coefficient of diffusion.

The accuracy of equation (19) is evaluated for some specific relationships between the coefficient of diffusion and concentration.

First consider a non-linear problem (11), (12) at

$$\bar{D}(c) = 1 - c. \tag{20}$$

Its solution is given in ref. [2] and yields the following expression of a diffusion flow:

$$j = \frac{0.332}{\sqrt{\tau}}. \tag{21}$$

On the other hand, substituting equation (20) into formula (19), one obtains

$$j = \frac{1}{\sqrt{(3\pi\tau)}} \approx \frac{0.326}{\sqrt{\tau}}. \tag{22}$$

Comparison of relations (21) and (22) shows that in the given case an error of the proposed approximate method is less than 2%.

Next consider the exponential dependence of the coefficient of diffusion on concentration

$$\bar{D}(c) = \exp\{\lambda(c-1)\}. \tag{23}$$

The solution of problem (11), (12), (23), obtained by a numerical method is given in ref. [3]. The following formula was also suggested in that book for a diffusion flow:

$$j = \frac{0.564}{1 + 0.177\lambda}\frac{1}{\sqrt{\tau}} \tag{24}$$

which 'operates' well within the range $-1.5 \leqslant \lambda \leqslant 3.5$.

Substituting equation (23) into the expression under the integral sign (19) gives

$$j = \left[\frac{2(\lambda + e^{-\lambda} - 1)}{\pi\lambda^2\tau}\right]^{1/2}. \tag{25}$$

Figure 1 presents a comparison of relations (25) and (24) (solid and dashed lines, respectively). It is seen that the maximum difference between these formulae amounts to around 4%.

Remark 1. Sometimes, instead of the exact non-linear equation (11) the following simplified linear equation can be used for approximate calculations:

$$\frac{\partial c}{\partial \tau} = \langle\bar{D}\rangle\frac{\partial^2 c}{\partial x^2} \tag{26}$$

where the constant $\langle\bar{D}\rangle$ is equal to the integral mean value of the diffusion coefficient

$$\langle\bar{D}\rangle = \int_0^1 \bar{D}(c)\,dc. \tag{27}$$

The solution of equation (27) with initial and boundary conditions (12) is determined by

$$c = \operatorname{erfc}\left(\frac{x}{2[\langle\bar{D}\rangle\tau]^{1/2}}\right) \tag{28}$$

where

$$\operatorname{erfc} z = \frac{2}{\sqrt{\pi}}\int_z^\infty \exp(-z^2)\,dz$$

and the corresponding diffusion flow has the form

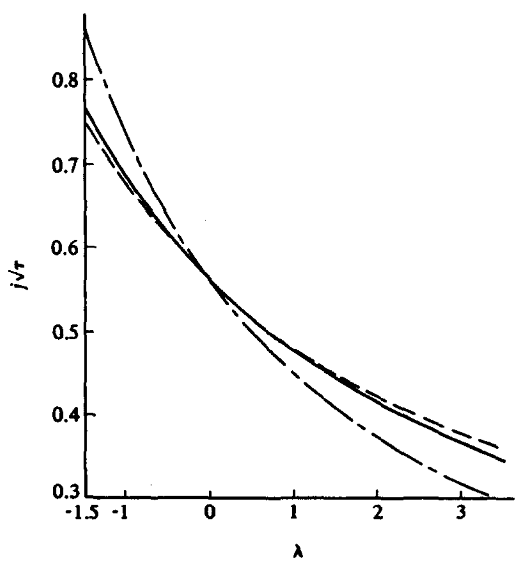

FIG. 1. Diffusion flow to the wall for exponential dependence of the coefficient of diffusion on concentration (23): ——, numerical calculation [3], equation (24); – – –, equation (25) obtained by the 'carry-over' method of integral transforms; –·–·–, equation (31) obtained by the method of averaging.

$$j = -\langle \bar{D} \rangle \left(\frac{\partial c}{\partial x} \right)_{x=0} = \left(\frac{\langle \bar{D} \rangle}{\pi \tau} \right)^{1/2}. \quad (29)$$

At the constant coefficient of diffusion expressions (19) and (29) coincide and give an accurate result. The validity of the approximate equation (29) will be checked on specific examples which were considered above. For linear dependence of the coefficient of diffusion on concentration (20), equation (29) gives

$$j = \frac{1}{\sqrt{(2\pi\tau)}} \approx \frac{0.399}{\sqrt{\tau}}. \quad (30)$$

Comparison of expressions (21) and (30) shows that in this case the procedure of averaging equation (27) leads to an error of about 20%. Therefore, the accuracy of the earlier obtained equation (22) is an order of magnitude higher than that of equation (30).

Substituting the exponential expression (23) into formula (29), with regard for (27), yields

$$j = \left(\frac{1 - e^{-\lambda}}{\pi \lambda \tau} \right)^{1/2}. \quad (31)$$

Calculations by this formula are presented in Fig. 1 by a dashed-dotted line. It is seen that the maximum error of equation (31) amounts to about 15%. Therefore, the accuracy of the carry-over method of integral transforms is in this case 3.5 times more accurate than the averaging method.

3. NON-STATIONARY MASS TRANSFER COMPOUNDED BY A VOLUMETRIC CHEMICAL REACTION

Consider now mass transfer between a wall and a quiescent medium in which there proceeds a chemical reaction with the rate $F_v = F_v(C)$. In dimensionless variables the corresponding non-stationary problem is formulated as

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} - f_v(c) \quad (32)$$

$$\tau = 0, c = 0; \quad x = 0, c = 1; \quad x \to \infty, c \to 0$$

where

$$c = \frac{C}{C_s}, \quad x = \frac{X}{a}, \quad \tau = \frac{Dt}{a^2}, \quad f_v(c) = \frac{a^2 F_v(C)}{DC_s}.$$

In the case of the nth-order reaction, corresponding to the relation $F_v = K_v C^n$, where K_v is the volumetric reaction rate constant, it should be assumed in equation (32) that

$$f_v(c) = k_v c^n \quad (33)$$

where $k_v = a^2 K_v C_s^{n-1}/D$.

The approximate solution of non-linear problem (33) makes use of the Laplace–Carson transform (3). The carrying over of the integral operator under the sign of the function f_v according to rule (2) will give the ordinary differential equation

$$\frac{d^2 u}{dx^2} = pu + f_v(u) \quad (34)$$

$$x = 0, u = 1; \quad x \to \infty, u \to 0. \quad (35)$$

The introduction of the new variable w by equation (15) makes it possible to reduce the order of equation (34). This gives the equation with separable variables the integral of which has the form

$$2 \int_0^u [pu + f_v(u)] \, du = w^2, \quad w = \frac{du}{dx}. \quad (36)$$

In deriving this expression, the boundary condition at infinity (35) was taken into account.

Making use of equation (36) and boundary condition (35) at $x = 0$, the derivative on the wall surface is calculated as

$$\left(\frac{du}{dx} \right)_{x=0} = -\left[p + 2 \int_0^1 f_v(u) \, du \right]^{1/2}. \quad (37)$$

Applying the inverse Laplace–Carson transform to both parts of equation (37), find the diffusion flow $j = -(\partial c/\partial x)_{x=0}$

$$j = (\pi\tau)^{-1/2} \exp(-\xi\tau) + \xi^{1/2} \operatorname{erf}(\xi\tau)^{1/2} \quad (38)$$

where

$$\xi = 2 \int_0^1 f_v(c) \, dc. \quad (39)$$

The approximate relation (38), (39) provides an exact asymptotic result at small and large values of the dimensionless time τ.

For the nth-order reaction it should be assumed in equation (38) that

$$\xi = \frac{2k_v}{n+1}. \quad (40)$$

It can be easily verified that in the limiting cases $k_v \to 0$ and $k_v \to \infty$ expressions (39) and (40) lead to the accurate result. Moreover, for the first-order reaction, $n = 1$, equations (38) and (40) are exact.

4. MASS TRANSFER WITH A VOLUMETRIC REACTION WITH THE COEFFICIENT OF DIFFUSION BEING ARBITRARILY DEPENDENT ON CONCENTRATION

Consider the equation

$$\frac{\partial c}{\partial \tau} = \frac{\partial}{\partial x} \bar{D}(c) \frac{\partial c}{\partial x} - f_v(c) \quad (41)$$

with initial and boundary conditions (12), where

$$f_v(c) = a^2 F_v(C)/[C_s D(C_s)]$$

the rest of the dimensionless quantities are introduced in the same way as in equation (11) at $C_0 = 0$.

Making use of the method described in Section 1, pass over in problem (41), (12) to the concentration image u. Integrating the resulting ordinary differential equation taking into account the attenuation at infinity yields

$$2\int_0^u [pu\bar{D}(u)+f_v(u)\bar{D}(u)]\,du = \left[\bar{D}(u)\frac{du}{dx}\right]^2.$$

Taking the rest of this equality at $x = 0$ and using the inverse Laplace–Carson transform, the following approximate expression can be found for a diffusion flow:

$$j = \left[\frac{2}{\pi\tau}\int_0^1 c\bar{D}(c)\,dc\right]^{1/2} \exp(-\zeta\tau) + \zeta^{1/2}\,\mathrm{erf}\,(\zeta\tau)^{1/2} \quad (42)$$

where

$$\zeta = \frac{\int_0^1 f_v(c)\bar{D}(c)\,dc}{\int_0^1 c\bar{D}(c)\,dc}.$$

At $f_v = 0$, this formula changes over to equation (19), whereas at $\bar{D} = 1$ it changes over to equations (38) and (39).

5. MASS TRANSFER FROM A DROPLET TO A FLOW WITH THE COEFFICIENT OF DIFFUSION BEING ARBITRARILY DEPENDENT ON CONCENTRATION

Consider the external problem of non-stationary mass exchange between a droplet (bubble) and a steady-state flow. It is assumed that concentration in the liquid at the initial time instant is uniform and equal to C_0 whereas on the droplet surface it is constant and equal to C_s. The equation of non-stationary mass transfer in a solid phase with allowance for the dependence of the coefficient of diffusion on concentration can be written in the form

$$\frac{\partial c}{\partial \tau} + Pe\,(\mathbf{v}\cdot \mathrm{grad}\, c) = \mathrm{div}\,[\bar{D}(c)\,\mathrm{grad}\, c] \quad (43)$$

where $Pe = aU/D(C_s)$ is the Peclet number, a the characteristic dimension of a droplet (radius for a spherical droplet), U the characteristic velocity of flow (liquid velocity at a distance from a droplet for a translational flow); the rest of the dimensionless quantities are introduced in the same manner as in equation (11). The initial and boundary conditions for equation (43) are similar to equation (12) where the value $x = 0$ corresponds to the droplet surface.

In ref. [4] it is shown that at large Peclet numbers (in the diffusion boundary layer approximation) the solution of the problem on mass exchange between droplets and bubbles with a flow (43), (12) should be sought in the form

$$c = c(z, \Psi) \quad (44)$$

where Ψ is the dimensionless stream function. The new variable $z = z(\tau, \theta)$ (θ is the coordinate along the droplet surface) can be selected so that the unknown function (44) will be determined by solving problem (11), (12), where τ and x should be substituted by z and Ψ. It is important to emphasize that the variable z is independent of the function $\bar{D}(c)$.

Allowing for the above and using the results of Section 2 the following approximate formula for the mean Sherwood number is obtained:

$$\frac{Sh(\bar{D},\tau)}{Sh(1,\tau)} = \left[2\int_0^1 c\bar{D}(c)\right]^{1/2}. \quad (45)$$

Here, $Sh(\bar{D},\tau)$ is the mean Sherwood number with the coefficient of diffusion being arbitrarily dependent on concentration, $\bar{D} = \bar{D}(c)$, and $Sh(1,\tau)$ is the mean Sherwood number at the constant coefficient of diffusion, $\bar{D} = 1$. It will be recalled that for spherical droplets and bubbles in an axisymmetric flow the mean Sherwood number is determined as

$$Sh(\bar{D},\tau) = -\frac{1}{2}\int_0^\pi \sin\theta \left[\bar{D}(c)\frac{\partial c}{\partial r}\right]_{r=1} d\theta$$

where c is the solution of problem (43), (12) at $x = r-1$; r and θ are the dimensionless (related to the droplet radius) radial and angular coordinates.

For translational and axisymmetric shear flows past a spherical droplet it should be assumed in equation (45), according to ref. [5], that

$$Sh(1,\tau) = \left[\frac{Q\,Pe}{\pi(\beta+1)}\coth\left(\frac{Q\,Pe}{\beta+1}\tau\right)\right]^{1/2} \quad (46)$$

$$Q = \begin{cases} 2/3 \text{ for a translational flow,} \\ \quad Pe = aU_\infty/D(C_s) \\ 3 \text{ for a shear flow, } Pe = a^2G/D(C_s) \end{cases}$$

where a is the droplet radius, U_∞ the liquid velocity at infinity, G the shear coefficient, and β the ratio of dynamic viscosities of the droplet and the surrounding medium ($\beta = 0$ corresponds to a gas bubble).

6. NON-STATIONARY DIFFUSION COMPOUNDED BY A SURFACE REACTION

A non-linear problem on non-stationary mass exchange between a quiescent liquid and a wall on the surface of which there proceeds a heterogeneous chemical reaction with the rate $F_s = F_s(C)$ is studied. It is assumed that at the initial time instant the concentration in the liquid volume is constant and equal to C_0. The corresponding equation, initial and boundary conditions for the concentration are written in the form

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} \quad (47)$$

$$\tau = 0, c = 0; \quad x = 0, \frac{\partial c}{\partial x} = -f_s(c); \quad x \to \infty, c \to 0 \quad (48)$$

where the dimensionless variables are introduced in the following way:

$$c = \frac{C_0 - C}{C_0}, \quad x = \frac{X}{a}, \quad \tau = \frac{Dt}{a^2}, \quad f_s(c) = \frac{aF_s(C)}{DC_0}.$$

Making use of the Laplace–Carson transform (3), taking into account rule (2), obtain the ordinary differential equation with the non-linear boundary condition

$$\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} = pu \tag{49}$$

$$x = 0, \quad \frac{\mathrm{d}u}{\mathrm{d}x} = -f_s(u); \quad x \to \infty, u \to 0. \tag{50}$$

The general solution of the equation with constant coefficients (49), which satisfies the conditions of damping at the infinity (50), is given by

$$u = u_s \exp(-p^{1/2} x) \tag{51}$$

where $u_s = u_s(p)$ is the image of the surface concentration which should be determined in the course of problem solution.

Substituting expression (51) into boundary condition (49) at $x = 0$ yields the non-linear algebraic equation to determine u_s

$$p^{1/2} u_s = f_s(u_s). \tag{52}$$

Using the Laplace–Carson inverse transform [1] equation (52), with the correspondence $f(u_s) \simeq \mathbf{L} * f(c_s)$ taken into account, the following integral equation for the surface concentration is obtained:

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \int_0^\tau \frac{c_s(\lambda)\,\mathrm{d}\lambda}{(\tau - \lambda)^{1/2}} = f_s(c_s). \tag{53}$$

This equation can be integrated numerically. The diffusion flow is recalculated in terms of the surface concentration by

$$j = f_s(c_s) \tag{54}$$

which is the consequence of the non-linear boundary condition (48) on the wall surface at $x = 0$.

It is important to emphasize that the non-linear integral equation (53) obtained with the aid of the approximate method is exact for the arbitrary relation $f_s = f_s(c)$. The proof of this statement will be given.

Application of the Laplace–Carson transform to linear equation (47), with regard for initial condition (48), gives the equation for image (49). The solution of this equation, which damps at infinity, is given by formula (51). Hence, the diffusion flow image is defined as

$$\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)_{x=0} = -p^{1/2} u_s. \tag{55}$$

Applying the Laplace–Carson inverse transform to both parts of this equality, the relationship between the derivative on the wall and surface concentration is obtained

$$\left(\frac{\partial c}{\partial x}\right)_{x=0} = -\frac{\mathrm{d}}{\mathrm{d}\tau} \int_0^\tau \frac{c_s(\lambda)\,\mathrm{d}\lambda}{(\tau - \lambda)^{1/2}}.$$

Eliminating, with the aid of this expression, the quantity $(\partial c/\partial x)_{x=0}$ from the non-linear boundary condition on the wall surface (48), the same equation (53) is obtained, as was to be shown.

Remark 2. The solutions obtained by the 'carry-over' method of integral transforms can be used as the first approximation in numerical computer calculations which are based on interation methods.

REFERENCES

1. V. A. Ditkin and A. P. Prudnikov, *Integral Transforms and Operation Calculus*. Izd. Nauka, Moscow (1974).
2. P. Ya. Polubarinova-Kochina, *The Theory of Phreatic Water Flow*. Izd. Nauka, Moscow (1977).
3. J. Crank, *The Mathematics of Diffusion*. Clarendon Press, Oxford (1975).
4. A. D. Polyanin, Method for solution of some non-linear boundary-value problems of non-stationary diffusion-controlled (thermal) boundary layer, *Int. J. Heat Mass Transfer* **25**(4), 471–485 (1982).
5. A. D. Polyanin and V. V. Dil'man, New methods of the mass and heat transfer theory—1. The method of asymptotic correction and the method of model equations and analogies, *Int. J. Heat Mass Transfer* **28**(1), 25–43 (1985).

LA METHODE DE TRAITEMENT DES TRANSFORMEES INTEGRALES DANS DES PROBLEMES NON LINEAIRES DE TRANSFERT DE CHALEUR ET DE MASSE

Résumé—Une méthode approchée simple est suggérée pour résoudre des problèmes non linéaires de transfert variables de chaleur et de masse. La méthode garantit la représentation précise des conditions aux limites qui conduit à un comportement asymptotique correct pour les grands temps et qui donne un résultat précis sur les problèmes linéaires. On étudie le transfert thermique variable entre une paroi et un milieu en repos, avec une conductivité thermique dépendant arbitrairement de la température. On considère des problèmes non-linéaires de transfert de masse avec des réactions volumiques et surfaciques d'ordre quelconque.

DIE METHODE DER "ÜBERTRAGUNG" VON INTEGRALER TRANSFORMATIONEN BEI NICHT-LINEAREN PROBLEMEN DER WÄRME- UND STOFFÜBERTRAGUNG

Zusammenfassung—Es wird eine einfache Näherungsmethode zum Lösen von nicht-linearen Problemen der instationären Wärme- und Stoffübertragung vorgeschlagen. Die Methode garantiert die genaue Erfüllung von Anfangs- und Randbedingungen, sie führt häufig zu einer zutreffenden Asymptote bei großen Zeiten und erzielt ein genaues Ergebnis für lineare Probleme. Die instationäre Wärmeübertragung zwischen einer Wand und einem ruhenden Medium mit beliebig temperaturabhängiger Wärmeleitfähigkeit wird untersucht. Ferner werden instationäre Probleme der Stoffübertragung mit volumetrischen und Oberflächenreaktionen jeglicher Ordnung untersucht.

МЕТОД ПРОНОСА ИНТЕГРАЛЬНЫХ ПРЕОБРАЗОВАНИЙ В НЕЛИНЕЙНЫХ ЗАДАЧАХ МАССО- И ТЕПЛОПЕРЕНОСА

Аннотация—Предлагается простой приближенный метод решения нелинейных задач нестационарного массо- и теплопереноса. Метод обеспечивает точное выполнение начального и граничных условий, часто приводит к правильной асимптотике при больших временах и дает точный результат для линейных задач. Рассмотрен нестационарный теплообмен стенки с неподвижной средой при произвольной зависимости коэффициента теплопроводности от температуры. Исследованы нестационарные задачи массопереноса с объемной и поверхностной реакцией любого порядка.